Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Some Observations on the Mechanism of Entrainment

P. M. Bevilaqua* and P. S. Lykoudis†
Purdue University, Lafayette, Ind.

I. Introduction

ALTHOUGH entrainment of fluid from the surroundings is fundamental to the development of turbulent free shear flows, the actual mechanism of entrainment is not well understood. The fundamental process is the diffusion of turbulent vorticity; irrotational fluid that becomes rotational is said to have been entrained. Various descriptions of this process have been suggested, but these can be reduced to two basic hypotheses. Entrainment is due either to the viscous diffusion of vorticity at the smallest scales of the turbulence[1-3] or to the action of large-scale "mixing jets" that engulf volumes of fluid in bulk.[4,5] We have prepared this Note to outline our reasons for rejecting the small-scale mechanism. From considerations of similarity, we argue in the next section that a viscous diffusion mechanism is inconsistent with the observed self-preserving development of many flows. Some experimental evidence for the existence of a large-scale mechanism is described in the last section.

II. Surface Layer Hypotheses

The existence of a well-defined boundary is characteristic of free turbulent flows. Across the greater part of the flow, the intensity of the turbulence is relatively constant. Near the boundary, however, the turbulent fluctuations diminish abruptly, resulting in a sharp interface between the regions of turbulent and nonturbulent motion. This interface is a continous but highly convoluted surface whose irregular motion produces the intermittently turbulent signal from a fixed probe near the flow boundary.

On the basis of measurements in a plane wake, a round jet, and the boundary layer on a flat plate, Corrsin and Kistler suggested that the interface is a thin fluid layer in which the turbulence is diffused by viscosity. The characteristics of the surface layer, or superlayer, were inferred by a mixture of intuition and observation. Since, Corrsin and Kistler reasoned, the rate of vorticity production is proportional to the vorticity already present, once a fluid element has been sheared by viscous stresses at the boundary, its vorticity is rapidly increased. Thus, the interface remains sharp. Because the turbulence in the superlayer exists in a condition of equilibrium between diffusion by viscosity and amplification by turbulent straining, Corrsin and Kistler saw it as being similar to the turbulence in Kolmogorov's universal equilibrium range of the energy spectrum, which is balanced between dissipation at higher wave numbers and amplification at lower wave numbers. They concluded, by analogy, that the scale of the turbulence at the boundary and the thickness of superlayer must therefore be small, on the order of Kolomogorov's dissipation length

$$\lambda = (\nu^3/\Phi)^{1/4} \tag{1}$$

where Φ is the rate at which turbulent energy is dissipated in the equilibrium range and ν is the viscosity of the fluid.

Furthermore, Corrsin and Kistler reasoned that the average rate of advance for such a diffusing surface of vorticity depends only on the fluid viscosity and the intensity of the vorticity fluctuations, ω. The appropriate combination of these parameters, giving the speed at which the surface advances, was surmised to be

$$V_s = (\nu\omega)^{1/2} \tag{2}$$

The vorticity of turbulence in the equilibrium range must be proportional to $(\Phi/\nu)^{1/2}$, and so the speed of advance becomes

$$V_s = (\nu\Phi)^{1/4} \tag{3}$$

which is the Kolmogorov velocity scale.

The superlayer hypothesis has had a continuing influence on the study of the interface and the mechanism of entrainment. However, some of the underlying assumptions have been questioned. Moffatt,[6] for example, pointed out that the dissipation of vorticity is also proportional to the vorticity already present; on these grounds the boundary is as likely to be diffuse as sharp. Phillips[3] argued that the speed at which the surface advances may be more characteristic of the velocities of the energy containing eddies, although he retained the notion of a small-scale entrainment process.

The suggested equivalence between the turbulence in the superlayer and the turbulence in the equilibrium range raises a more fundamental question. Turbulence in the universal equilibrium range is, by definition, independent at conditions in the mean flow (except for the energy supplied); yet the speed at which the interface advances must adjust both to the flow geometry—wakes diffuse more rapidly than jets, for example—and to changes in the spreading rate as each flow decays. With an entrainment velocity determined by properties of the small-scale turbulence, the observed self-preserving development of many flows is not possible. This can be shown from considerations of similarity.

In the region of self-preservation, the entrainment velocity must be proportional to the scale velocity of the mean flow, U_c. According to Kolmogorov's hypothesis the dissipation is proportional to rate at which energy is supplied, $\Phi = U_c^3/L_c$, where L_c is the length scale of the mean flow. The requirement for self-preserving diffusion at the Kolmogorov velocity is

$$(\nu U_c^3/L_c)^{1/4} \sim U_c \tag{4}$$

In general, the self-preserving scale parameters vary as $U_c \sim x^{-m}$ and $L_c \sim x^n$; the requirement for self-preservation then becomes $m=n$. This is only true for the cases of the plane wake and round jet.

An assumption that the entrainment velocity is determined by simple diffusion of the energy containing eddies leads to the same result. In self-preserving flows, the vorticity of these eddies is proportional to U_c/L_c, and the requirement for spreading at the diffusion velocity is that

$$(\nu U_c/L_c)^{1/2} \sim U_c \tag{5}$$

Presented as Paper 75-115 at the AIAA 13th Aerospace Sciences Meeting, Pasadena, Calif., Jan. 20-22, 1975; received Jan. 27, 1975; revision received April 29, 1977.

Index category: Jets, Wakes, and Viscid-Inviscid Flow Interaction.

*Presently Supervisor, Research and Engineering Department, Rockwell International, Columbus Aircraft Division. Member AIAA.

†Professor and Head, School of Nuclear Engineering, Purdue University. Associate Fellow AIAA.

The requirement again becomes $m=n$. Thus, except possibly in the plane wake and the round jet, another mechanism must adjust the entrainment velocity to the self-preserving rate of development.

III. Large Eddy Hypothesis

We believe that the entrainment is determined by the large, coherent eddies whose permanence has only recently been discovered.[7,8] Although it was known that large-scale motions of the interface are correlated with an increase in the rate of entrainment,[9] it was presumed that this motion controls the entrainment indirectly, by increasing the surface area of the interface. The origin of the motion was not recognized until the coherent eddies were discovered, and various hypotheses regarding instabilities of the mean flow and the turbulence were suggested to account for the large-scale motions.

The mechanism of entrainment suggested by recent studies of the wake[10] and mixing layer[11] is illustrated in Fig. 1. The motion of the surface, which is the exposed side of a large, rotating eddy, displaces the surrounding fluid much as air is displaced by a swelling ocean wave. There is an interface for the turbulent vorticity that is kept sharp by the action of the large eddy in bringing high-intensity turbulence to the surface from the interior of the flow. At the interface, viscosity diffuses the turbulent vorticity, thus retarding the motion of the external flow relative to the surface. The decelerated fluid is then entrained into the turbulent core of the flow by the rotation of the large eddy. The newly entrained fluid is not immediately diffused by the other scales of the turbulence but continues to rotate with the large eddy.

This large eddy mechanism is consistent with earlier observations and, indeed, accounts for them. The convection of turbulent fluid to the surface and newly entrained fluid into the core would appear as a process of engulfment. In fact, Grant[4] reported an approximate periodicity in the occurrence of the "mixing jets" and associated a circular motion with each of them. The interface is kept sharp by this same rotation of the large eddies, rather than by the process of gradient steepening supposed by the superlayer hypothesis. Finally, the adjustment to the requirements of self-preservation is accomplished by the large eddies which are in what Townsend called a state of moving equilibrium, increasing in scale at the same rate as the mean flow diffuses.

In view of the significant role the large vortices have in the development of the mean flow, it is perhaps surprising that the permanence of these eddies was not detected sooner, especially in the wake of the cylinder, which has been so much studied. However, as seen in the photographs of Papailiou and Lykoudis,[12] the effect of these vortices is to cause a periodic variation in the direction of the mean velocity vector. Such variations cannot be detected by a single hot-wire sensor parallel to the cylinder axis, since the sensor responds to changes in the magnitude, but not the direction, of the velocity vector.

To substantiate the visual evidence in these experiments[12] the U and V components of velocity in the wake of a glass cylinder 0.635 cm in diam and 45 cm long were measured with a Thermo-Systems linearized anemometer and an X probe. The wind tunnel used was of the open return type; the test section has a circular cross section 45 cm in diam and 360 cm long. Figure 2 shows the turbulence components on the wake centerline 50 diam behind the cylinder, where the flow was generally considered to be fully turbulent. The Reynold's number is 10,000. Although the U component does appear to be completely turbulent, the V component clearly shows periodic oscillations. These oscillations correspond to the shedding frequency; that is, they occur at the first subharmonic. The anemometer signal was also analyzed with a Nelson-Ross spectrum analyzer. These spectra are also shown in the figure. The subharmonic is apparent in the spectrum of the V component, but its existence is masked in the U component. The range of this behavior was not established, due to limits on the speed of the tunnel and length of the test section, but similar oscillations were seen as far as 350 diam behind the cylinder and at Reynolds numbers up to 70,000.

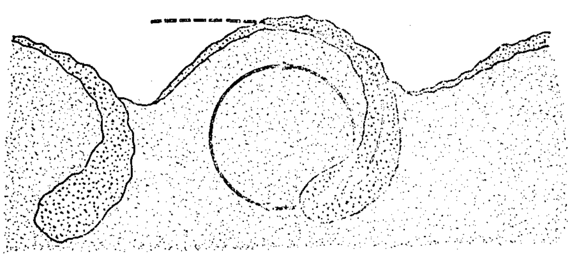

Fig. 1 Entrainment occurs by the enfolding action of the large eddies within the surface "waves".

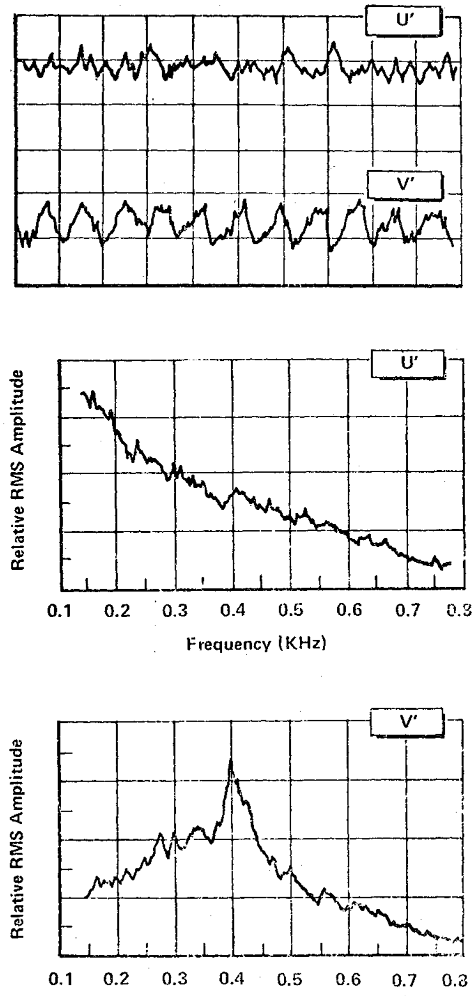

Fig. 2 Spectra of the fluctuating velocity components in the wake of a cylinder at $Re=10{,}000$ and $x/d=50$.

IV. Conclusions

From a critical re-examination of the assumptions and implications of the superlayer hypothesis, we have concluded that entrainment is a process that more nearly resembles a folding of the turbulent and nonturbulent fluids by the rotation of the large eddies. This mechanism is suggested by recent observations of the coherent eddies in wakes and mixing layers and provides a unified explanation of the nibbling and engulfing mechanisms that have been proposed.

References

[1]Corrsin, S. and Kistler, A. L., "Free Stream Boundaries of Turbulent Flows," NACA Rept. 1244, 1955.

[2]Townsend, A. A., "Mechanism of Entrainment in Free Turbulent Flows," *Journal of Fluid Mechanics*, Vol. 26, 1966, pp. 689-715.

[3]Phillips, O. M., "The Entrainment Interface," *Journal of Fluid Mechanics*, Vol. 51, 1972, pp. 97-118.

[4]Grant, H. L., "The Large Eddies of Turbulent Motion," *Journal of Fluid Mechanics*, Vol. 4, 1958, pp. 149-190.

[5]Bradshaw, P., "The Understanding and Prediction of Turbulent Flow," *Aeronautical Journal*, July 1972, pp. 403-418.

[6]Moffatt, H. K., "Interaction of Turbulence with Rapid Uniform Shear," SUDAER 242, also AD 626298, Stanford University.

[7]*Colloquium on Coherent Structures in Turbulence*, University of Southhampton, England, March 1974.

[8]*Proceedings of the Project Squid Workshop on Turbulent Mixing*, Purdue University, Lafayette, Ind., May 1974.

[9]Gartshore, I. S., "Experimental Examination of the Large Eddy Equilibrium Hypothesis," *Journal of Fluid Mechanics*, Vol. 24, 1965, pp. 89-98.

[10]Bevilaqua, P. M. and Lykoudis, P. S., "Mechanism of Entrainment in Turbulent Wakes," *AIAA Journal*, Vol. 9, Aug. 1971, pp. 1657-1659.

[11]Brown, G. L. and Roshko, A., "On Density Effects and Large Structures in Turbulent Mixing Layers," *Journal of Fluid Mechanics*, Vol. 64, 1974, pp. 775-816.

[12]Papailiou, D. D. and Lykoudis, P. S., "Turbulent Vortex Streets and the Mechanism of Entrainment," *Journal of Fluid Mechanics*, Vol. 62, 1974, pp. 11-31.

Laminar Boundary-Layer Flow over an Insulated Accelerating Slender Wedge

Cz. M. Rodkiewicz* and J. Skiepko†

The University of Alberta, Edmonton, Alberta, Canada

Introduction

WINGS with the cross section of a narrow wedge commonly are used as lifting surfaces in high-speed vehicles. This work deals with one such problem. A slender two-dimensional wedge, shown in Fig. 1, moving at a high Mach number, is subjected to a nonimpulsive change in its speed. The time history of the flowfield is analyzed. Boundary-layer equations are solved by the method suggested by Moore.[1] It is assumed that the flow in the potential region is quasisteady.

The disturbances due to the presence of the wedge are confined within a curved shock, beyond which the freestream conditions prevail. In this study, we assume the shock to be a plane. For a slender wedge with a sharp leading edge, the shock is attached at the leading edge. Thus the flows on the two sides of the wedge become independent of each other and can be considered separately. Thus the problem under investigation may be reduced to the problem of finding the unsteady flow over a flat plate after an increase or decrease in its speed, and subject to the possible freestream variations in the thermodynamic properties.

Problem Formulation

A slender infinite wedge is subjected to a motion in which every point of the wedge has the same velocity vector parallel to the plane of symmetry of the wedge. Initially the wedge velocity is constant, $U=U(t_0)$. At time t_0 the wedge begins, let us say, to accelerate and reaches a new constant velocity, $U=U(t_1)$, at time t_1. During the time interval $t_0 \leq t \leq t_1$, the wedge velocity $U(t)$ is some function of time.

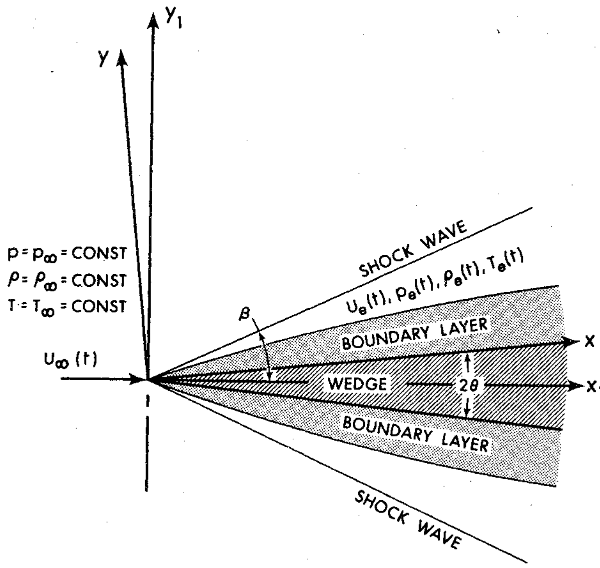

Fig. 1 Schematic of the flow configuration.

The flowfield between the shock and the boundary layer, in our formulation, is potential. In spite of this somewhat simplified configuration, even the potential part of the flow is difficult to solve analytically. We assume further that outside the boundary layer the flow is in a quasisteady state. This assumption is useful, provided that $U'(t)$ is not too large, $U(t)$ is large, and when the distance x from the edge of the wedge is small enough (primes indicate derivatives).

Some of the flow parameters are indicated in Fig. 1. Having $U_\infty(t)$, pressure p_∞, density ρ_∞, and temperature T_∞, we can calculate U_e, p_e, ρ_e, and T_e by the use of Rankine-Hugoniot relations. As the velocity $U(t)$ varies with time, the position of the shock also will change with time, and our calculations will be good if the shock velocity relative to the wedge is small. In any case, there exists a neighborhood around the edge of the wedge where this condition may be satisfied to as close a degree as we desire; i.e., there always will be a region where our procedure is applicable. Having U_e, p_e, ρ_e, and T_e, we can compute the flow parameters within the boundary layer.

Rankine-Hugoniot Conditions

In order to find the flow parameters between the boundary layer and the shock wave, we used the Rankine-Hugoniot relationships, which, when simplified, yield

$$\frac{U_e}{a_\infty} = \left\{ \left[\frac{(\gamma-1)\zeta+2}{(\gamma+1)\zeta} \right]^2 \zeta - \zeta + M_\infty^2 \right\}^{1/2} \tag{1}$$

$$\frac{p_e'}{p_e} = \frac{2\gamma}{\gamma+1+2\gamma(\zeta-1)} \frac{d\zeta}{dt} \tag{2}$$

$$\frac{T_e}{T_e'} = \frac{(\gamma+1)^2\zeta+2(\gamma-1)(\zeta-1)(\gamma\zeta+1)}{2(\gamma-1)[\gamma(\zeta-1)+\gamma(\zeta+1)/\zeta]d\zeta/dt)} \tag{3}$$

$$\zeta = M_\infty^2 \sin^2\beta \tag{4}$$

Received Aug. 25, 1976; revision received May 16, 1977.

Index category: Supersonic and Hypersonic Flow.

*Professor.

†Post-Doctoral Fellow on leave from the University of Warsaw, Poland.